Mark Mitchell
35201: Cases in Financial Management
Section 81: Spring 2023

Real Options Analysis

PREVIEW

The Discounted Cash Flow, or DCF concept of converting cash flows over several periods into a value as of a specific date is a powerful concept in finance. The resulting NPV generates a value that management can use to decide whether to invest or reject an investment project under consideration and maximize shareholder wealth.

Despite the benefits of using DCF to evaluate investment decisions, it was not until the 1970s before large corporations began to embrace the use of DCF. They had historically favored some measure of an accounting rate of return, or a simple payback technique. Today, over 75% of large corporations use DCF metrics such as the Internal Rate of Return, or IRR, and Net Present Value, or NPV, in their investment decisions. Interestingly though, over half of large corporations still employ the payback method as a tool to help them evaluate and select projects in which to invest.

While DCF and NPV are powerful finance concepts and widely used in practice, one drawback is that a standard DCF analysis assumes the project will progress as intended, given the assumptions at the beginning of the project. In other words, as it is typically implemented, DCF is a static model. In reality, outcomes don't always occur as expected; the optimal path will often change before the project ends. Just because a result doesn't happen as expected doesn't mean that the assumptions were incorrect; rather, expected values are based on the known data and associated probabilities at the time.

Management has the option to alter a project after it has commenced. It can choose to expand the project, abandon the project, or modify the project in numerous ways. These opportunities to alter projects down the road are referred to as real options. Moreover, they can be valued, often with considerable difficulty, at the beginning of the project. The real options may end up altering the timing and the size of the project relative to the initial NPV analysis.

The purpose of this lecture note is to introduce the concept of a real option. I'll start with a refresher of the option terminology and basic option pricing models. Next, I'll describe how real investment decisions can have embedded options, which we label as real options, and I'll follow with a few examples. I'll close with an overview of how real options impact overall firm valuations, especially in high-growth firms. But first, let's start with a case involving the Decision Tree Corporation.

Management of Decision Tree is analyzing Bang-Bang, a project which requires an upfront investment of \$100 million, with perpetual cash flows commencing at the end of the year. Assume the future cash flows are \$12 million per year, with a 40% probability, or \$6 million per year, with a 60% probability. Think of the \$12 million outcome as the good state of the world and the \$6 million outcome as the bad state of the world. The cost of capital for Bang-Bang is 8.0%, and the risk-free rate is 3.5%. The NPV for Bang-Bang is:

$$\boxed{\text{Eq. 1}} \qquad \text{NPV} = \$5 \text{ million} = -\$100 \text{ million} + \frac{\left[\$12 \text{ million } (0.40) + \$6 \text{ million } (0.60)\right]}{0.08}$$

Employing basic NPV analysis, management of Decision Tree accepts the proposed Bang-Bang project and invests immediately.

Suppose Decision Tree can delay the investment by one year, at which time management will know whether the macroeconomic issues have resulted in a good outcome -- the good state of the world, or the bad state of the world. Assume as well that once management knows which state of the world the project is in, it will remain in that state into perpetuity. Thus, if the Bang-Bang project is undertaken in the bad state of the world where the expected cash flows are \$6 million, then all subsequent periods will also occur in the bad state of the world.[1] In this case, management will estimate the NPV at the end of the year as:

$$\boxed{\text{Eq. 2}} \qquad \text{NPV} = -\$28.5 \text{ million} = -\$103.5 \text{ million} + \frac{\$6 \text{ million}}{0.08}$$

Since the NPV is negative in the bad state of the world, management would reject investing in Bang-Bang if the investment decision were delayed for a year. Before we estimate the NPV at the end of the year for the good state of the world, note the underlying assumption behind the cost of the investment if delayed to the end of the year. Rather than assuming the investment cost remains fixed at \$100 million, the calculation assumes instead that it will increase at the risk-free rate, as commodity costs, labor costs, overhead, etc., are expected to rise as prices increase.

In the good state of the world, the NPV at the end of the year is:

$$\boxed{\text{Eq. 3}} \qquad \text{NPV} = \$46.5 \text{ million} = -\$103.5 \text{ million} + \frac{\$12 \text{ million}}{0.08}$$

Here, the NPV is large relative to the size of the investment, which shouldn't be surprising given that

[1] Note that this does not imply that the cash flows will be exactly \$6 million each year; that would imply risk-free cash flows. Rather, once it is known that the project is in the good or the bad state of the world, the expected cash flows will either be \$12 million or \$6 million, as opposed to expected cash flows of \$8.4 million, which is the case at the beginning of the project.

Bang-Bang ended up being launched in the good state of the world.[2] Management of Decision Tree Corporation will recommend moving forward on the project due to the +NPV. As outlined above, we show investment decisions at two different points in time, today and one year from today. Given a choice to invest in Bang-Bang at the current time, management will recommend moving forward on the project, given the NPV of $5 million. But this decision assumes the investment is either made at the current time or not at all. We also show that if Decision Tree can wait and make the investment at the end of the year, management will recommend acceptance in the good state of the world, and rejection in the bad state of the world, based on the respective NPVs.

The next step is to compare the two investments, each of which generates a positive NPV, that is, an NPV of $5 million at the current time, or an NPV of $46.5 million at the end of the year if investing in the good state of the world. Both are attractive in isolation. But what if they are viewed as mutually exclusive projects? To make this comparison, we need to consider them at the same point in time. We know the NPV of investing immediately is $5 million. But what is the NPV today of waiting one year, and then choosing to invest in the good state of the world? It is:

$$\boxed{\text{Eq. 4}} \qquad \text{NPV} = \$15.6 \text{ million} = \left[\frac{-103.5 \text{ million}}{1.035} + \frac{\$12 \text{ million}}{0.08(1.08)}\right]0.40$$

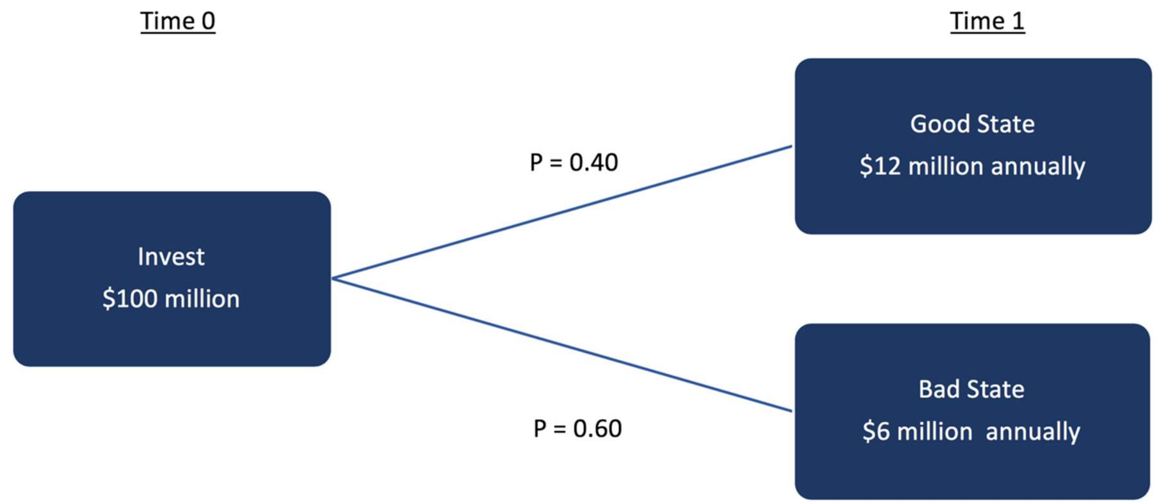
There are several points to note about the various components of the above NPV calculation. First, the investment cost increases by 3.5% if the decision is delayed by one year. Thus, we assume an investment cost of $103.5 million in year one and note the discounting back to today at the discount rate of 3.5%. In present value terms, the cost of the investment today is identical to the cost of investing one year later. Second, since the potential future cash flows of $12 million commence in two years, given the one-year delay of the investment decision, the value of the perpetuity is as of the end of the first year, and thus must be discounted another year back to the current year. Third, the formula reflects the fact that the probability in the current period of realizing the good state of the world one year later is equal to 40%.

While the NPV of immediately investing in Bang-Bang is positive at $5 million, the NPV of the alternative project -- delaying the investment for one year and then investing in the good state, if it occurs, is over three times greater, at $15.6 million. We have deliberated framed these as mutually exclusive projects, in keeping with the way that we have applied NPV and DCF analysis so far, which has been conducted in a static framework. But given there are future decisions that can alter the profitability of the project, we can model this project a bit differently using probability tree analysis and decision tree analysis. We will end up at the same place, but the approach will be different and will yield valuable insights into understanding more complicated decisions faced by management.

The starting point is a probability tree, as displayed below.

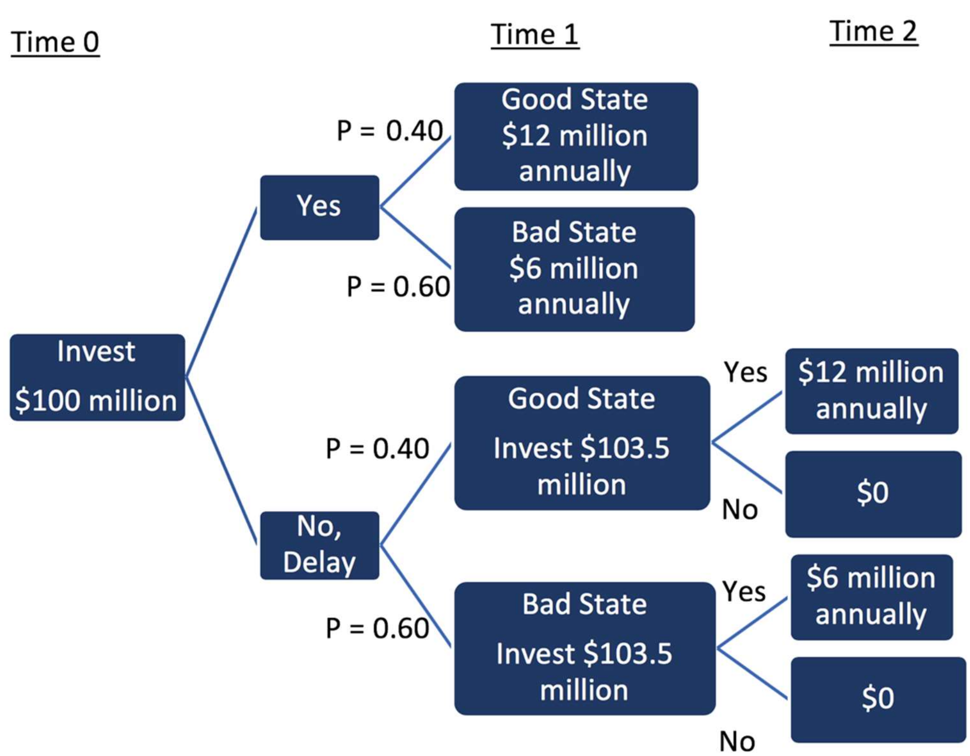
[2] Note that in both scenarios, we assume an 8% discount rate, but this estimate doesn't have to be the same for both states of the world.

Bang Bang Project: Probability Tree



There is no new information in these diagrams; it is just a different way of illustrating the analysis. I label it a probability tree, since there are no decisions made along the way. The decision will be made to invest now or not at all based on the cash flows provided. As shown above, the associated NPV is $5 million. The purpose of starting with the probability tree is to lay the groundwork for the decision tree in the illustration below.

Bang Bang Project: Decision Tree



The decision tree illustrates the options associated with the Bang-Bang project. Management can either decide to invest today, or instead to wait and reevaluate in one year. By waiting one year, management learns which state of the world exists, good or bad. In both cases, management has the option at the end of the first year to invest $103.5 million in Bang-Bang, but will optimally choose to invest only in the good state. As we saw, the NPV of investing today is $5 million and the NPV of delaying the project and investing a year later, if in the good state of the world, is $15.6 million. Thus, the ability to delay the project by one year increases its value by $10.6 million. Hence the option to delay is worth $10.6 million. Indeed, the delay option is more than twice the value of the original NPV. Without this option to delay the project, management bears the risk of finding itself in the bad state of the world, with a negative NPV.[3]

Decision tree analysis is merely an extension of NPV, rather than an alternative to NPV. Basic NPV doesn't allow for contingent decisions throughout a project by management. Thus, decision tree analysis delivers considerable value to the NPV framework by incorporating the concept of subsequent decisions and the resulting optionality. However, decision tree analysis has its drawbacks. First, it is not easy to estimate the probabilities associated with the various states of the world. Second, the risk is changing as the project moves through the tree. In our illustration, we assume risk and expected cash flows in the respective states of the world do not change, but these are overly simplistic assumptions, at least when it comes to real options in the real world. Now, let's turn to the basics of option pricing to gain insight into how we can value real options, like the option to delay the Bang-Bang projects, similar to the valuation of financial options with which many of us are familiar.

BASIC OPTION TERMINOLOGY

Before delving more deeply into real options, it is crucial to understand the basic option terminology: payoff diagrams, standard option valuation models, etc.[4] The subsequent sections provide the basic framework necessary to understand financial options, in terms of what is most relevant for corporate finance. It is a cursory review of what you learned in *Investments.*

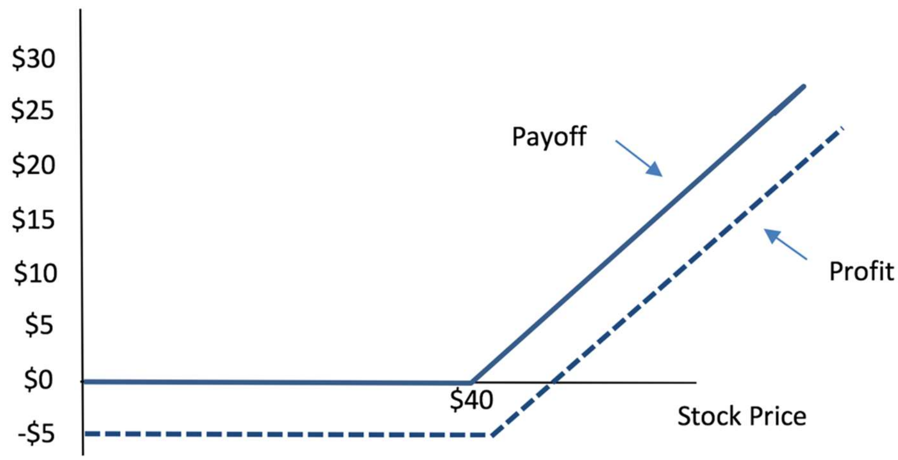
There are two basic types of options, calls and puts. A call option gives the owner the right to purchase an asset by a specified date (an exercise or maturity date) at a set price, typically referred to as the exercise or strike price. A put option gives the owner the right to sell an asset by a specified date at a set exercise or strike price. Figure 1 shows the payoff (in blue line) and profit (in dotted blue line) diagrams for the owner of a call option on the option exercise date at an exercise price of $40. Assume the option was purchased several weeks earlier at $5. Further assume these options are European options and thus can only be exercised on the exercise date (American options can be exercised on any date prior to the exercise date).[5]

[3] But note that choosing to delay and investing in the good state of the world does not guarantee success. Rather, the expected future cash flows are double. The Bang Bang project can still end up with a negative outcome – it just has a lower probability of doing so.

[4] This refresher will also be useful when we discuss a firm's capital structure as a portfolio of option contracts and when we cover convertible securities.

[5] Most options traded are American options, but European options are a bit easier to analyze and the properties of the

Figure 1



If the asset price exceeds $40 on the exercise date, the holder of the option will exercise the option as it will be "in-the-money" – above the exercise price. And if the asset price is equal or less than $40, the holder allows the call option to expire, and it becomes worthless. The call option value at expiration is:

Eq. 5 $\quad C = \max [S - K, 0)$

where C = call value, S = asset price, and K = exercise price. Suppose the asset price is $60 on the exercise date. The holder will exercise and realize a profit of $15 ($60 - $40 - $5). It is important to note that the mere exercise of the option when the asset price exceeds the exercise does not imply the call option holder realizes a profit on the investment. For example, if the asset price is $41 on the exercise date, the holder will exercise and incur a loss of $4 ($41 - $40 - $5). It is more beneficial to realize a loss of $4 via exercising the call option, versus a loss of $5 if the holder chooses not to exercise it. As shown in the payoff and profit diagram, the call option will have positive value on the exercise date if the asset price exceeds $40, and the holder will profit from the investment if the asset price exceeds $45.

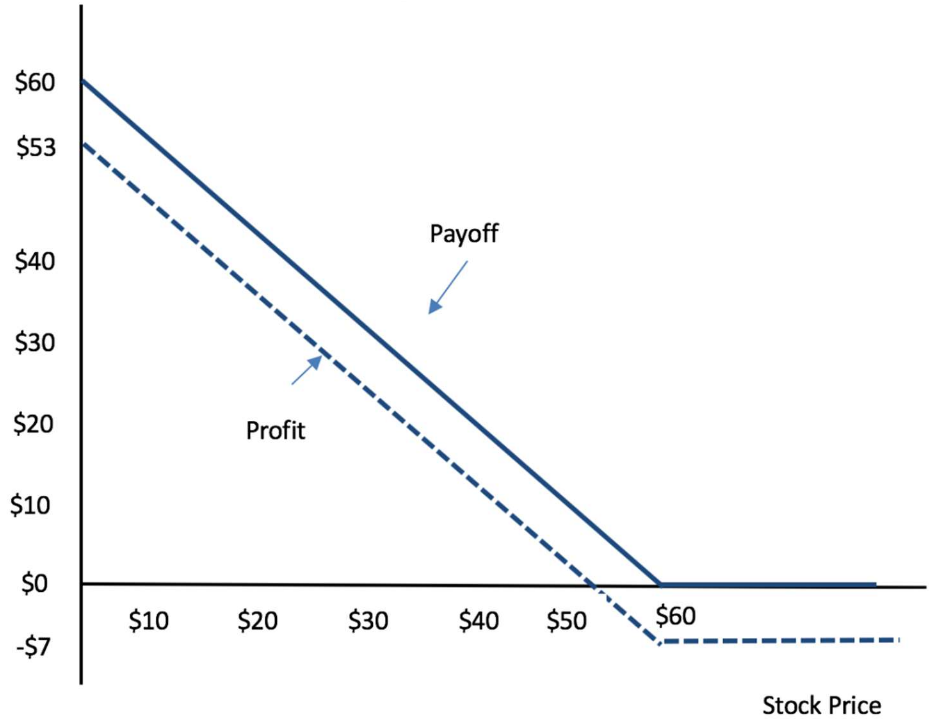
As indicated above, a put option gives the owner the right to sell the asset at a specified price. The put option value at expiration is:

Eq. 6 $\quad P = \max [K - S, 0]$

Thus, the holder benefits if the asset price is less than the exercise price on the date of exercise. Figure 2 shows the payoff (indicated by the solid blue line) and profit (indicated by the dotted blue line) diagrams for the owner of a put option as the option exercise date and an exercise price of $60.

two are quite similar. The possibility of early exercise makes American options more valuable, but early exercise is generally not optimal, as it extinguishes the remaining life of the option. Indeed, the option holder can usually obtain more value by selling it to another party than by exercising early. There are exceptions, however, such as if a large dividend will be paid on the underlying asset, which reduces the value of a call option.

Figure 2



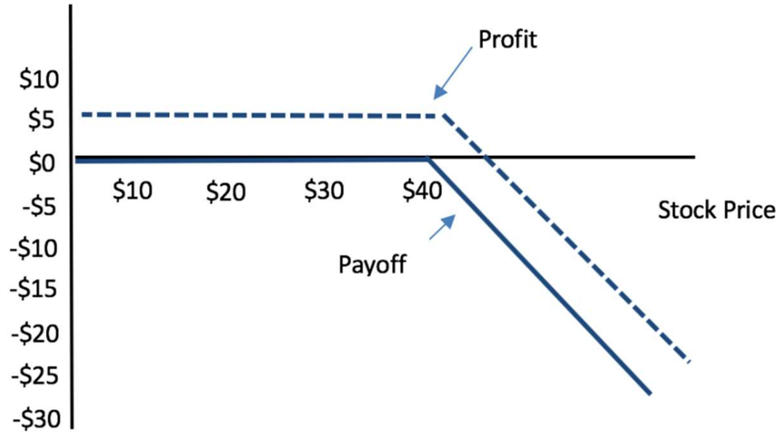
Assume the option was purchased several weeks earlier for $7. As illustrated, the option is “in the money” and thus has a positive payoff, if the asset price on the exercise date is less than $60. For example, if the asset price is $50 on the exercise date, the holder will exercise and realize a profit of $3 ($60 - $50 - $7). And as indicated with the call option, the holder of the put option will exercise at certain prices even it means losing money on the roundtrip trade. For example, if the asset price is $55 on the exercise date, the holder will exercise and realize an overall loss of $2 ($60 - $55 - $7). But exercising and realizing a loss of $2 is preferable to not exercising and realizing a loss of $7.

There are two sides to every option contract, the writer, or the seller of the option on the one side, and the party who bought the option on the other side. By selling the option, the writer receives a premium upfront, but is subject to subsequent liabilities. That is, the option writer never receives funds in addition to the upfront premium, as the holder of the option will not exercise the option if the option is out of the money. The option writer’s profit or loss is the exact opposite of the option buyer’s profit or loss, as it is a zero net-sum transaction. Thus, the payoff to the seller of a call option is:

Eq. 7 $$\text{Call Seller Payoff} = -\max [S - K, 0] = \min [K - S, 0]$$

The payoff, and profit, at expiration to the seller of the call option is shown below in Figure 3 and is the exact opposite of the payoff and profit diagram for the call buyer as shown in Figure 1.

Figure 3

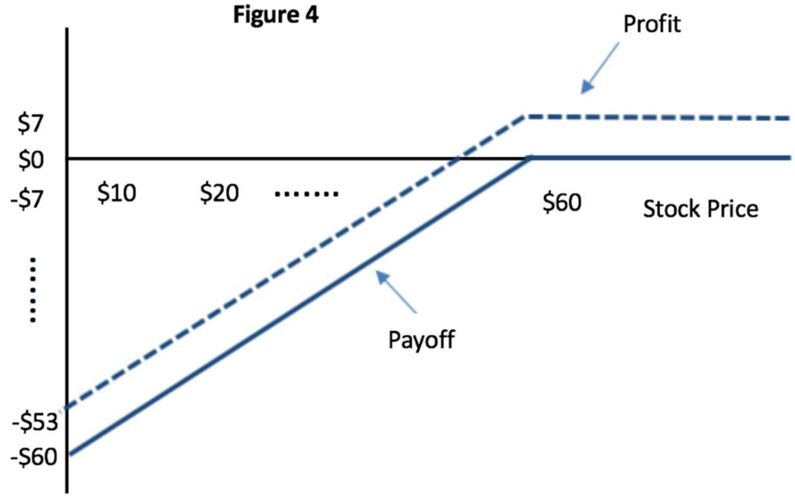


Finally, the payoff to the seller of a put option is:

Eq. 8 Put Seller Payoff = -max [K – S, 0] = min [S – K, 0]

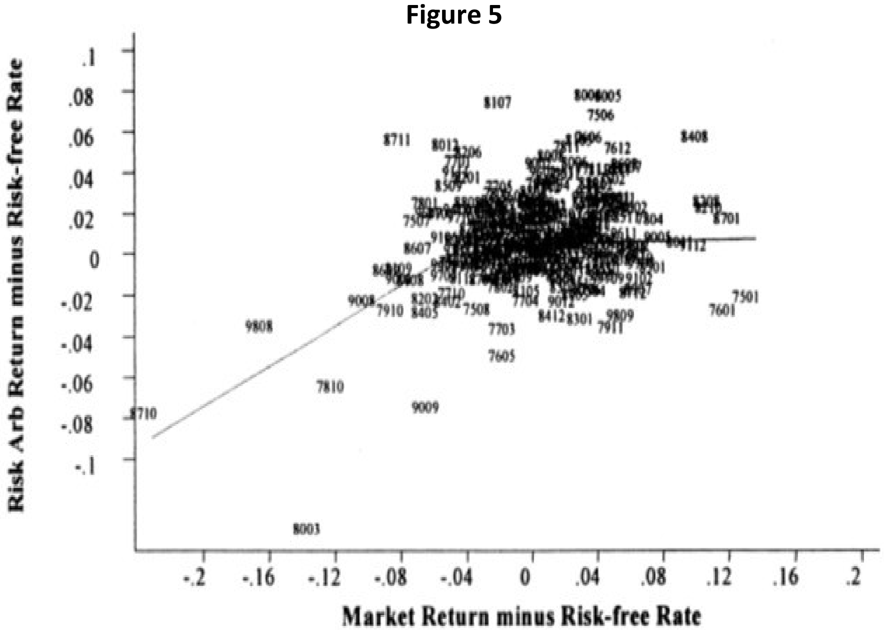
The payoff, and profit, at expiration to the seller of the put option is shown below in Figure 4 and is the exact opposite of the payoff and profit diagram for the put buyer as shown in Figure 2.

Figure 4



The payoff to the put writer is similar to that of certain investment strategies including that of merger arbitrage. Todd Pulvino and I were the first researchers to document that hedge fund strategies such as merger arbitrage have a payoff structure comparable to that of sellers of out-of-the-money S&P 500 index put options, in 2001. See Figure 5 below, which is from our original paper and covers the period 1975-1998.

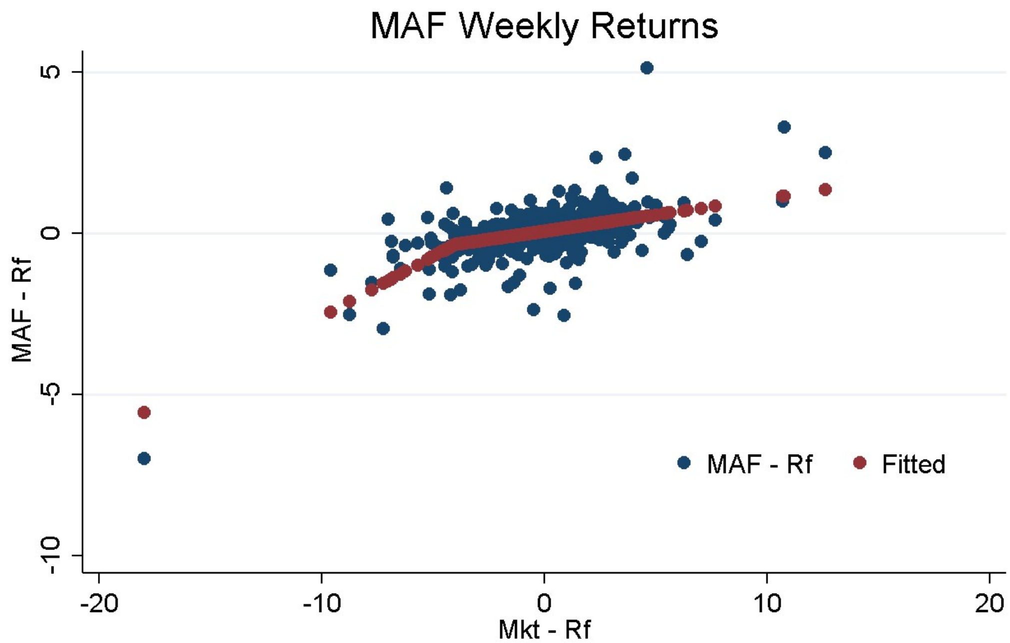
Figure 5



Below, see Figure 6, which covers the 2001-2017 period in support of the original analysis.[6] The beta tends to be somewhat flat when the market is increasing, flat or slightly declining, but increases substantially when the market is tanking. Overall, our research shows that similar to out-of-the-money index put options, a small premium is collected in most states of the world by merger arbitrageurs, but occasionally a hefty payout is made, just like with insurance policies. Moreover, while investment managers are typically judged against a CAPM or a multi-factor asset pricing model such as Fama-French, risk arbitrage may be better evaluated relative to a replicating portfolio with a non-linear option-like payoff.

[6] Mitchell, Mark, and Todd Pulvino, "Characteristics of Risk and Return in Risk Arbitrage," *Journal of Finance*, 2001.

Figure 6



The purpose of illustrating the above is to point out that if an investment strategy looks like an option, then it should be modeled and valued as an option. The same is true for real options, as discussed at the end of this lecture note.

FACTORS DRIVING OPTION VALUES

The value of an option is driven by six primary factors:

(1) asset price

Increases in the asset price result in higher call option values, as do decreases in the asset price of the put option.

(2) strike price

Increases in the strike price result in lower call option values, and higher values in put options. Some companies which have realized steep stock price declines will reset the strike prices of way out-of-the-money employee stock options to "better align incentives," often to the chagrin of shareholders.

(3) time to expiration

Longer-dated options, both calls and puts, have higher values than short-dated options, everything else constant. eBay's stock price is currently $45.98. A put option with a strike price of $42.50 and expiring in one day is worth about 1 cent. But if the put option has two years of remaining life, the value is about $6.20 -- 620 times higher.

(4) *volatility of underlying asset*
If volatility increases unexpectedly, the probability of the asset performing either extremely well or extremely poorly increases, and thus leads to higher values for both calls and puts. For example, if Kinder Morgan, a pipeline transportation and energy storage firm with long-term fixed contracts, were to expand into exploration and production, its volatility would likely increase, resulting in higher value for both call and put options.

(5) *risk-free interest rate*
Since the exercise price is not paid until a future date, increases in the interest rate will increase the value of call options and decrease the value of put options, everything else held constant.

(6) *expected dividends*
As discussed in the *Dividend Policy* lecture note, dividends reduce the stock price on the ex-dividend date. Thus, dividends paid prior to the exercise date have the effect of reducing the price of call options and increasing the price of put options. Note that dividends can lead to early exercise of call options depending on the size of the dividend.

THE BINOMINAL OPTION PRICING MODEL

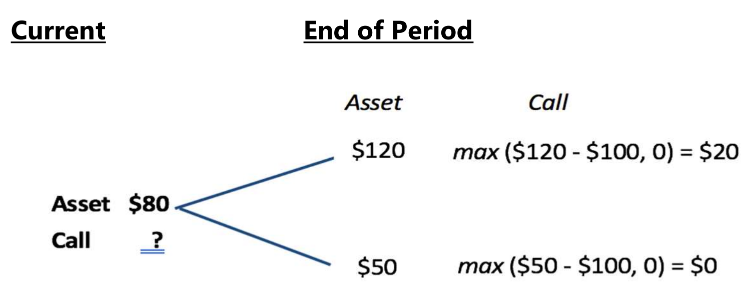
When one thinks of option pricing models, what first comes to mind is the famous Black-Scholes model published in 1973.[7] Fischer Black began attempting to price options in 1968, started collaborating with Myron Scholes in 1969, and circulated the first version of their paper in 1970. As with many innovative articles, Black and Scholes had great difficulty in publishing their option pricing model. Three academic journals rejected their article before it was finally accepted by the *Journal of Political Economy,* based on the strong recommendations of Eugene Fama and Merton Miller.[8] In the same year, Robert Merton published his option pricing model independent of Black-Scholes; often their joint model is referred to as Black-Scholes-Merton.[9] I discuss their model in the next section. While the Black-Scholes-Merton model is mathematically complicated, a far easier way to price options was developed a few years after, namely the binomial option pricing model which converges to the Black-Scholes-Merton model with enough time steps.

As described below, the binomial option pricing model is a simple, yet extremely powerful model that can be employed to solve very complex option pricing problems.[10] Whereas the Black-Scholes-Merton

[7] Black, Fischer and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* (1973). Fischer was on the faculty of Chicago Booth during 1971-1975. Myron was an MBA and Ph.D. student at Booth during the 1960s, and later joined the faculty of Chicago Booth from 1973-1981.
[8] The *Journal of Political Economy* is a famous economics journal edited at the Kenneth Griffin Department of Economics, University of Chicago.
[9] Merton, Robert, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* (1973).
[10] Cox, Ross, and Rubinstein are largely credited with the development of the binomial option pricing model. Cox, John, Stephen Ross, and Mark Rubinstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*

model requires advanced mathematics such as solving partial differential equations, the binomial model only requires basic algebra and is exceptionally intuitive. As implied by the name, the binomial option pricing model makes the simple assumption that the asset price has only two possible values at the end of the next period. Like the Black-Scholes-Merton model, the binomial model constructs replicating portfolios of a risk-free bond and the underlying asset which generates the same payoff as that of the option. And given the law of one price -- that two assets which have the same payoff should have the same price -- it can be shown from the model that the value of the option is equivalent to the cost of setting up the replicating portfolio. A simple example follows below.

Consider a call option that expires in one period and has an exercise price of $100. The current price of the underlying asset is $80 and the interest rate during the period is 5%. At the end of the period, the asset price will either be $120 or $50. All of this information can be summarized on the binomial tree below which provides the timeline and all of the possible outcomes.



As the timeline above shows, the current asset price is $80 and there are two possible outcomes at the end of the period. First, the asset price can increase to $120; we refer to this state as the *up* state. Second, the asset price can decrease to $50; we refer to this state as the *down* state. We know that at the *up* state asset price of $120, the holder of the call option will choose to exercise the option at the strike price of $100, thereby yielding a payoff of $20. At the *down* state asset price of $50, the option holder will opt not to exercise the option and thus the payoff will be $0.

Now that we know the two payoffs to the call option in both states or outcomes, that is, $20 and $0, the next step is to create a portfolio of the underlying asset and a risk-free bond which will exactly match the payoffs to the option in the two states of the world. Looking ahead, we can compute the cost of the replicating portfolio, which gives us the value of the option due to the law of one price. Think of the risk-free bond as cash, and either investing to earn the risk-free rate or borrowing at the risk-free rate. In the *up* state of the world, the replicating portfolio is:

Eq. 9 $$\$120\Delta + 1.05B = \$20$$

The replicating portfolio in **Eq. 9** of the asset and a risk-free bond gives the same payoff of $20 as the call option in the *up* state. There are two unknowns, Δ and B. Δ refers to the number of shares of the asset in the replicating portfolio and B refers to the number of risk-free bonds in the replicating portfolio. Assume

(1979). Ross died unexpectedly in 2017, otherwise would have received a Nobel Prize in Economics for his work on arbitrage price theory and option pricing models.

the price of each risk-free bond is $1. There is some combination of asset shares and risk-free bonds which delivers the same payoff as the call option. But using basic algebra, we cannot solve for the amounts of assets and bonds, since we have two unknowns and only one equation.

In the *down* state, the replicating portfolio is given as:

Eq. 10 $\quad \$50\Delta + 1.05B = \$0$

Now, there are two simultaneous equations, **Eq. 9** and **Eq. 10**, with two unknowns, Δ and B, and thus we can solve for the answer, that is, the number of asset shares and risk-free bonds. By subtracting **Eq. 10** from **Eq. 9**, we obtain:

Eq. 11 $\quad \$70\Delta = \$20$

Solving for Δ yields:

Eq. 12 $$\Delta = \frac{\$20}{\$70} = 0.2857$$

The replicating portfolio will own 0.2857 shares of the asset. Now that we know the solution for Δ, we can plug the solution, 0.2857, into either one of the two equations to solve for B as shown below by plugging into the *up* state equation:

Eq. 13 $\quad \$120\ (0.2857) + 1.05B = \$20$

And yielding:

Eq. 14 $\quad B = -\$13.605$[11]

By borrowing $13.605 and simultaneously purchasing 0.2857 shares of the asset, this portfolio yields the same payoff of $20 in the *up* state as shown above in **Eq. 13** and the exact same payoff of $0 in the *down* state as shown below in **Eq. 15**:

Eq. 15 $\quad \$50\ (0.2857) + 1.05B = \$0$

Since the portfolio of 0.2857 shares of the asset purchased via borrowing $13.605 yields the same payoff in both outcomes or states of the world as the payoff to the call option (again in both states of the world), then the price of the call option must equal the cost of constructing this replicating portfolio. And the value of the replicating portfolio as of time 0 is simply equal to the value of the asset minus the funds borrowed:

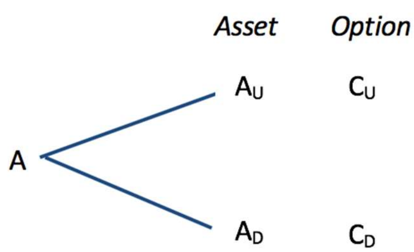
Eq. 16 $\quad \$80\ (0.2857) - \$13.605 = \$9.25$

[11] In case you have purged bad memories of Algebra I with respect to solving for unknowns in simultaneous equations, a quick refresher is available on www.khanacademy.org in the "Systems of Equations" module of Algebra I, specifically the section "Equivalent Systems of Equations and the Elimination Method" and the sub- section "Systems of Equations with Elimination: King's Cupcakes." And if you were captivated by the cupcake video, then try the potato chip video to figure out Arbegla's dilemma of under and over-ordering potato chips.

Since the value of the replicating portfolio is $9.25 as in **Eq. 16** above, then due to arbitrage, that is, equivalent securities must sell for the same price, the price of the call option must also be equal to $9.25. This result is pretty cool. That is, if you merely know the current asset price, the two possible asset prices at a future date, the exercise price, and the risk-free rate, you can compute the value of a call option without knowing the probabilities associated with the two future states of the world and without knowing the expected return on the underlying asset. And if the price of the call option is less (greater) than $9.25, arbitrageurs will buy (write) the call option and simultaneously short (purchase) 0.2857 shares of the asset and invest (borrow) $13.605 of cash, guaranteeing a riskless profit until the prices converge.[12]

The above stylized example can easily be generalized to apply to any option. As before, assume there is only one period, which has a beginning and end of the period and can be represented below.

Current **End of Period**



As shown, the current asset price is A, and it can increase to A_U or decrease to A_D. And the corresponding option prices are C_U and C_D. As before, the bond is given as B, and it earns (or pays) the risk-free (R_F) rate of interest. To compute the value of the option today, we calculate the number of units, Δ, in the asset and the number of bond units to create a replicating portfolio which has the same payoff as the option when the asset goes up or down. As shown earlier with the stylized example, here we solve generally for the two unknowns in the two simultaneous equations with the replicating portfolio.

Eq. 17a $$A_U \Delta + B(1 + R_F) = C_U$$

Eq. 17b $$A_D \Delta + B(1 + R_F) = C_D$$

Subtracting the C_D equation from the C_U equation and then solving for Δ yields:

Eq.18 $$\Delta = \frac{C_U - C_D}{A_U - A_D}$$

The Δ captures the sensitivity of the value in the option to changes in the underlying asset price. And then solving for B yields:

[12] Suppose that instead we were valuing a put option with a strike price of $100 and a current asset price of $80. If you work this one out on your own, the value of the put is $24.49.

$\boxed{\text{Eq.19}}$ $$B = \frac{C_D - A_D\Delta}{1+R_F}$$

or $$B = \frac{C_U - A_U\Delta}{1+R_F}$$

It follows, as we showed earlier, that the value of the option is simply equal to the cost of the replicating portfolio:

$\boxed{\text{Eq. 20}}$ $$C = A\,\Delta + B$$

With these three simple equations, **Eq. 18**, **Eq. 19**, and **Eq. 20**, one can calculate the value of an option with the binomial option pricing model. Note that if we plug in the assumptions from the stylized example discussed earlier, we obtain the same value for the option as shown below:

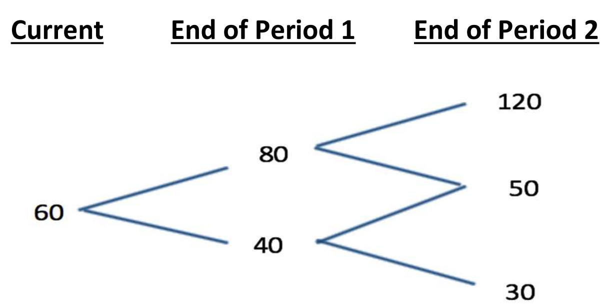
$\boxed{\text{Eq. 18a}}$ $$0.2857 = \frac{\$20 - \$0}{\$120 - \$50}$$

$\boxed{\text{Eq. 19a}}$ $$-\$13.605 = \frac{\$0 - \$50\left(0.2857\right)}{1 + 0.05}$$

$\boxed{\text{Eq. 20a}}$ $$\$9.25 = \$80\ (0.2857) - \$13.605$$

Thus, with this general formula, we obtain the exact same answer as earlier when we solved the specific simultaneous equations for the two unknowns.

Notwithstanding the powerful finding that pricing an option is straightforward, as demonstrated above, we have done so in a vacuum. The real world contains many more possible states (outcomes) and many more periods than our simplistic assumptions. It is conceptually easy to add more periods and we can demonstrate by adding another period to the example used earlier. Thus, we add another period to the previous example as shown below.



As shown above, the asset can either go up or down in each period. With two periods, there are four possible stock price outcomes at the end, rather than the two stock price outcomes in the one-period model. Assume the current asset price is \$60, rather than the previously assumed \$80, the exercise price remains at \$100, and the risk-free rate remains at 5%. To calculate the value of an option in a multi-period model, we start at the far right of the binomial tree and work backward. Note that the option expires at the end of the second period, and in only one state in this example -- that is when the asset price is \$120, when it will be optimal to exercise the option. At the end of the second period, the call option is worth

$20 when the asset price is $120, and is worthless for the other states where the asset price is $50 and $30.

Now that we know the values of the option for each of the states at the end of the second period, we work back and compute the value of the option at the end of the first period. The case where the asset price at the end of the first period is $80, with the two possible payoffs of $120 and $50, should be familiar since it is the exact binomial tree we solved earlier. Thus, the value of the option is $9.25 at the end of the first period where the asset price is $80.

In the case where the asset price drops to $40 at the end of the first period, this is easy to solve since neither asset price in the second period exceeds $100, as they are $30 and $50, respectively. Thus, with the asset price of $40, a call option with a strike price of $100 would have zero value, since there is zero probability that the ending asset price would exceed $100.

Now that we know the value of the call option in either state of the world at the end of the first period, we can work backward and determine the current value of the call option.

Eq. 18b $$0.23125 = \frac{\$9.25 - \$0}{\$80 - \$40}$$

Eq. 19b $$-\$8.81 = \frac{\$0 - \$40\,(0.23125)}{1 + 0.05}$$

Eq. 20b $$\$5.07 = \$60\,(0.23125) - \$8.81$$

Thus, the initial value of the call option is $5.07 given the exercise price of $100 and the current asset price of $60. Note how the Δ changes from the first period to the second period if the asset price increases to $80, that is, the Δ increases from 0.23 to 0.29. We will rehedge our position at each period as the risk changes; at the limit, this is referred to as dynamic hedging.

The purpose of showing the two-period example is to illustrate that the math employed is identical to that of the one-period example; it is just employed more times. And to make the binomial model realistic, we increase the number of periods and thus let the time between each period be very close -- each period could be one nanosecond. As the number of periods near infinity, the binomial option model approaches the Black-Scholes-Merton model, which we will consider next.

THE BLACK-SCHOLES-MERTON OPTION PRICING MODEL

A bit of history is important to understand the timing of the incredible breakthrough by Black-Scholes-Merton in developing the option pricing model.[13] Fischer Black began work as a consultant in the operations research group at Arthur D. Little, Inc. in Boston and was influenced by another consultant,

[13] This history is largely summarized from notes I have from Fisher Black.

Jack Treynor, in the same group. Treynor is known as one of the developers of the CAPM.[14] Though trained as a mathematician, Black had a strong interest in finance and economics and began to study the CAPM and other financial models while at Arthur D. Little. He attempted to use the CAPM to price securities other than stocks and began to focus on the valuation of warrants in 1968. Black recognized quickly that one couldn't use the CAPM to price warrants as the discount rate changes dynamically due to the non-stop changes in the riskiness of warrants as stock prices change and as time passes. Thus, Black took the novel step of attempting to price a warrant where its price depended on the underlying stock price and a host of other factors. He used the CAPM to account for every moment in the warrant's life, reflecting every possible stock price. The result was a differential equation with just one solution. However, Black was unable to solve the equation to obtain the solution. So, he quit working on it. But before temporarily quitting he recognized that the warrant value did not depend on the expected return of the stock; instead, it depended on the total risk, i.e., volatility of the underlying stock.

After receiving his Ph.D. from Chicago Booth, Scholes took a job as an assistant professor at MIT. He reached out to Black who continued to work at Arthur D. Little, nearby. It was pointed out to them that their differential equation can be written in the form of a famous heat equation with which high-level math students are familiar. Black hadn't focused on differential equations while pursuing his Ph.D. in Math at Harvard and wasn't aware of the heat equation at the time. But once they became aware of it, Black and Scholes were able to model the Black-Scholes equation in the form of a heat equation. And finally, they had their answer on how to value a warrant. Meanwhile, Robert Merton was working independently on the same problem, and derived roughly the same answer, although doing so outside of a CAPM model. Black and Scholes had a difficult time publishing their paper; it was rejected by a few academic journals. It wasn't until Gene Fama and Merton Miller intervened and told the *Journal of Political Economy* that it should give the paper another look that it was finally accepted for publication.

Going back to the binomial model, if you let the time period collapse to zero and thus have an infinite number of periods, the binomial option pricing model in effect becomes the Black Scholes model, though they didn't derive it that way, as the binomial model wasn't created until 1979.

Since we went step by step in the binomial option pricing model to obtain the insight, below we show the solution to the differential equation which Black had difficulty in solving years ago:

Eq. 21 $$C = SN(d1) - pv(K)N(d2)$$

In plain English, this solution states that the value of a call option, C, is equal to the value of the underlying stock price, S, multiplied by a delta, N(d1), minus the present value of a risk-free bond that pays the exercise price, K, on the call option's maturity date, multiplied by a delta, N(d2). Conceptually, this is what we saw with valuing call options with replicating portfolios in the binomial option pricing model. We found the value of a call option was equivalent to the cost of a replicating portfolio which purchased some amount of shares based on the Δ, delta, using borrowed funds, that is, -B. All this formula is saying is that you can replicate an investment in a call option via a levered investment in the underlying stock.

[14] While Treynor developed the first version of the CAPM, he never published it; according to Fisher's notes, "Jack's papers were never published in part because he is a perfectionist and was never quite satisfied with them."

The terms N(d1) and N(d2) require further explanation. First, the function N(d) is the cumulative probability distribution function for a variable with a normal distribution which many of you have learned at various times in math and statistics classes. It is the probability that a normally distributed variable will be less than d.

Eq. 22
$$d_1 = \frac{\ln\left[\frac{S}{pv(K)}\right]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

Eq. 23
$$d2 = d1 - \sigma\sqrt{T}$$

The important takeaways from the Black-Scholes equations are that the value of a call option increases with the stock price and decreases with the exercise price, as expected. And it increases with the time to maturity and the assumed volatility. As indicated earlier, the binomial model has discrete time steps, but as each period becomes shorter and shorter, the time steps approach continuous time and the distribution approximates the well-known normal distribution. The value drivers are the same for Black-Scholes as for the binomial model. And the vastly important principle of replicating portfolios applies to both in the same way, that is, the replicating portfolio is created by buying a certain number of shares using borrowed funds. That is, it is self-financing.

REAL OPTIONS

Many projects and real investments have options embedded in them, and thus traditional discounted cash flow analysis will underestimate the value of the project. Examples of these real options include the following:

(1) Expanding into new products or markets at later stages of the project based on initial success.
(2) Terminating projects or reducing exposure if initial results are unfavorable.
(3) Delaying implementation of the investment.
(4) Adjusting the type of production as input prices change during the life of the project.

The value of a real option is that as managers, we can learn from what is going on as we begin to undertake the project or are during the project. And based on this experience and what we have learned, we can adjust the investment to increase the expected profitability or to decrease the expected losses. Put differently, the traditional discounted cash flow methodology is static. Yet we live in a dynamic world and thus need to be equipped as situations evolve and impact the overall project value.

It is relatively straightforward to identify when a project has an embedded real option. There are three basic conditions which must be met:

(1) There has to be an underlying project whose value is subject to change through time due to the realization of various outcomes over the life of the project.
(2) The real option has to have a contingent-like payoff dependent on a certain event

occurring. That is, the real option has to look like a regular financial option with which we are familiar.

(3) There has to be some exclusivity on the option; otherwise, it is simply an opportunity and doesn't have real economic value.

ILLUSTRATIONS OF REAL OPTIONS

Below, I provide two illustrations of real options. The first illustration involves ScrollMotion, a digital marketing services firm which assists corporations in developing interactive sales presentations. ScrollMotion is contemplating a major shift in strategy. It intends to transition from a services-based operating model to a subscriber-based platform which allows the end-user to develop the interactive sales presentations internally. The issue is whether ScrollMotion should move forward on the major strategy shift, even though the NPV appears to be negative. Does moving forward on the -NPV project allow ScrollMotion to subsequently expand and create shareholder wealth? The second illustration concerns FreshDirect, an online food grocer which has recently built a new operating plant. In this illustration, the focus is on the value of abandoning the new plant in a state of the world where it performs far below expectations.

Strategy Shift at ScrollMotion

Background

ScrollMotion was launched in 2008 to assist major corporations, especially in publishing, to take their print content to the interactive touch format on tablets, etc. Over the next few years, ScrollMotion consulted for numerous large corporations in their transition to using mobile devices in their outreach to customers. While ScrollMotion was able to assist corporations in creating interactive presentations with the client who needed an internal designer or developer, the process was extremely time consuming and often required large teams of developers at ScrollMotion. In addition, the marketing and sales outreach was an arduous task, as it was difficult to convince the client that it was worthwhile to make the switch, given the lengthy process involved.

In 2016, ScrollMotion was at a crossroads. The company had not grown nearly as quickly as its venture capital backers had expected. As a result, ScrollMotion's valuation plummeted, and existing investors were hesitant to continue funding it. A new investor group emerged, and along with some of the existing investors, the new mindset was to radically alter the business structure of the firm, transitioning from a services firm to a SaaS subscription-model based platform. Doing so would require a complete regutting of the entire business model, from technology to its sales effort. In effect, ScrollMotion would walk away from its services business and thus virtually eliminate its revenues for the next few years.

To implement the new direction ScrollMotion was taking, the new investor group recognized a different type of managerial expertise would be required. So, they brought in seasoned executives from Apple who had been advocates of ScrollMotion. ScrollMotion been producing content for large corporations, as well as building content for one of Apple's mobility programs for small businesses.

This new direction required a complete revamp of ScrollMotion's software, to enable it to be used to help businesses assemble and share authentic, customized content for sales presentations, portfolios, digital collateral, and advertising. This platform, labeled Ingage, would allow users to assemble content, with the aid of their company- approved libraries, to generate sales presentations. Whereas ScrollMotion initially focused on the tablet, Ingage would transition beyond the iPad or iPhone with touch-screen interface to desktop and laptop computers with keyboards.

ScrollMotion estimates the transition and redevelopment to Ingage will cost $60 million to develop. The company has a present value of $40 million.[15] Thus, Ingage has a NPV of -$20 million, and viewed in isolation, it should not be developed. But by developing Ingage, ScrollMotion will have the option to create Ingage-Plus for the masses, which will require limited knowledge of technology by the end user in creating marketing content on social media. Ingage Plus will require a substantially larger investment than Ingage, due to the need to create an infrastructure that is able to build content for even the most inexperienced user. For example, one of the intended features of Ingage Plus is the ability to convert a basic PowerPoint presentation to an interactive format, with rudimentary assistance from the client. Based on Scrollmotion's current assessment, Ingage Plus will cost $200 million to develop; it has a NPV of roughly -$50 million.

The valuation metrics for Ingage and Ingage Plus are displayed below.

Ingage		**Ingage Plus**	
Cost	Value	Cost	Value
60	40	200	150
NPV		NPV	
-20		-50	

Based on the above NPV estimates, ScrollMotion should reject both projects. But should the company go ahead with developing Ingage and assessing the customer response to the software, management will amass valuable information which will influence its decision to go forward or not with Ingage Plus. If the company considers only the expected cash flows, and then discounting back to the current cash flows, management should pass on Ingage and by definition, on Ingage Plus as well, since to invest in Ingage Plus, Scrollmotion must first develop Ingage. Yet while Ingage Plus looks like a bad investment today, this may not be the case if we are able to value the real option of Ingage Plus by investing in Ingage today.

Real Option Analysis of Ingage Plus

To examine the real option value of Ingage Plus to ScrollMotion, we can create a framework just like that of analyzing financial options, as described earlier. First, it is important that the project looks like a financial option -- that is, there must be a non-linear contingent payoff. In the case of Ingage Plus, the cash flow

---

[15] These estimates factor into the reduction of cash flows due to exiting the services business.

estimates as provided above assume that Ingage has been completely developed. Put differently, one could develop Ingage Plus without developing Ingage first, but the cost would be substantially higher, and thus the NPV even more negative. As set up, the Ingage Plus cash flows are contingent on the prior development of Ingage.

Recall that the following inputs are necessary to value an option: (1) asset price, (2) strike price, (3) volatility, (4) time to maturity, and (5) interest rate. The asset price is given as the sum of the expected discounted cash flows of the Ingage Plus project, which is $150 million. The strike price is the $200 million upfront cost of undertaking the Ingage Plus project. We can think of this as an out-of-the-money call option, where the strike price is higher than the underlying security price. From a corporate finance perspective, we can think of a -NPV project as being out of the money. Likewise, a +NPV project can be viewed as an in-the-money option. What is important to realize is, as we show below, out-of-the-money options have value -- just not as much as in-the-money options, holding everything else constant.

We view risk as a negative feature in valuing a project, specifically its systematic risk. But with real options, there is an upside feature to risk, in that uncertainty can be a driver of value, assuming of course that management is prepared to act opportunistically and take advantage of the uncertainty associated with a project. Ideally, we know the volatiliy associated with the project. As with financial options, we must estimate the forward-looking volatility, as we never know the actual volatility in advance.

For the purposes of this illustration, we will not go into great detail with respect to estimating volatility estimates for Ingage Plus. But conceptually we can employ estimates from publicly-traded firms which are equivalent twins to Ingage Plus. Since there is no pure twin to Ingage Plus, a reasonable choice would be to estimate standard deviations of stock returns from small single-product software development corporations. We will assume this estimate is 75%. which we will employ here. Alternatively, we could estimate the standard deviation from running Monte Carlo simulations of the various outcomes for Ingage Plus, but this is beyond the scope of this course.[16]

The time to maturity assumes that ScrollMotion has only two years to commence the development of Ingage Plus, if Ingage is a success or if positive information arrives indicating that Ingage Plus becomes a +NPV investment. Last, assume the risk-free rate is 3%.

The five inputs are summarized below:

1. Asset Price: present value of Ingage Plus = $150 million
2. Strike Price: cost of Ingage Plus = -$200 million
3. σ: standard deviation of Ingage Plus: 75% annual
4. T: time to maturity = 2 years
5. RF: interest rate = 3%

Using the above assumptions in a calculator of the Black-Scholes option pricing model, the value of the

[16] That is, one can run simulations of all the possible outcomes and associated probabilities, and thereby generate an estimate of the project's volatility. Running the Monte Carlo simulations is not a difficult task per se, rather it is beyond the scope of this course due to time constraints.

real option to invest in Ingage Plus is worth $50.6 million.[17] Again, without considering this option value, Scrollmotion management would be advised to reject the investment in Ingage, as it yields negative NPV of -$20 million. Even though Ingage Plus also has a negative NPV of -$50 million, at the current time, it has substantial option value, such that the overall investment today of $60 million in Ingage has an adjusted NPV of $30.6 million. That is,

Eq.24 Ingage Adjusted NPV = Ingage NPV + Ingage Plus Real Option

Eq. 24a $30.6 million = -$20.0 million + $50.6 million

As shown above, the extremely high level of uncertainty about Ingage Plus generates substantial option value with respect to investing in Ingage, even though Ingage appears to be a value-destroying investment, and even considering Ingage Plus in isolation appears to destroy value.

I wrote the above synopsis for the Spring Quarter 2019 class in Corporate Finance. ScrollMotion did move forward with Ingage. And as expected, Ingage was horribly negative in terms of NPV, far worse than expected. On the positive side, it did generate massive tax-loss carryforwards due to a recapitalization which wiped out all the junior security holders! The investors moved forward on Ingage Plus. By March 2020, Ingage Plus was also heading into the bad state of the world, despite having built incredible software which sales managers loved. The sales team was experiencing traction at conferences, but the growth was slower than expected. And with COVID-19, the concern was that considering the shutdown of physical industry conferences, Ingage Plus had little chance to grow and reduce its massive cash burn. But given that the sales personnel of its potential customers were also unable to travel, this actually created an extraordinary demand for the product. And as of 2023, ScrollMotion is on its way to profitability, despite numerous setbacks. Had COVID-19 not occurred, it is likely that investor fatigue would have forced a sale at a price far below the expected valuation.

Plant Expansion at FreshDirect

Background

FreshDirect, an online grocer founded in 1999 in New York City, was profitable from 2011 to 2017. It is the largest online grocer in New York City, with more than 50% of the market share. With expected revenue of about $800 million in 2018, FreshDirect had grown beyond its capacity in the Long Island City plant just across the East River from Manhattan. In July 2018, FreshDirect opened a 650,000 square foot facility in the South Bronx, which allows it to double the overall size of the business. The new state-of-the art facility has robotic pick towers, smart routing technologies and nine miles of conveyor belts.

Assume that when the new Bronx plant goes into production, the incremental value to FreshDirect is $100 million. Beyond the typical systematic risk of operating an online grocery, there is the idiosyncratic risk that the plant may not live up to expectations. To simplify, assume that the plant will either work as

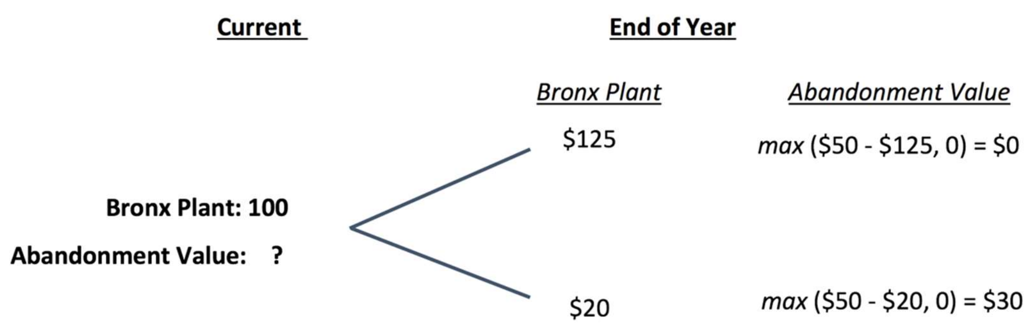
[17] Go to www.cboe.com/trading-tools/calculators and select Options Calculator.

intended or it will be an absolute failure. If the new plant works as intended, the value at the end of the year will be $125 million. That is the incremental value of the Bronx plant versus the old facility in Long Island City. But there is a low likelihood that the design and configuration of the new plant will give it only $20 million incremental value relative to the old facility, even after reconfigurations to improve the overall facility. In case the new Bronx plant is a complete failure, FreshDirect will have the option to sell it at the end of the year for $50 million (net of the cost of reopening the old facility in Long Island City) to a beer distributor.

Analysis

The ability of FreshDirect to sell the new plant at the end of the year for $50 million in case the new plant doesn't work as intended can be modelled as a real option. That is, the right for FreshDirect to sell the plant has real value, which we can refer to as abandonment value or disposal value. In financial derivative terms, it can be likened to a put option.

To facilitate our understanding of the treatment of the plant abandonment as a put option, all of the various assumptions are simplified. For example, only one period is specified. Likewise only two outcomes are specified, the most likely outcome, and the disaster state in which FreshDirect would sell the facility. We can use the binomial option pricing model described earlier to value the abandonment option of FreshDirect's Bronx facility. The information as provided can be summarized on the binomial tree below, which provides the timeline and possible outcomes.



As shown in the above timeline, the incremental value of the Bronx plant just before it opens for business is $100 million. Assuming that all goes well, the facility's incremental value will increase to $125 million at the end of the period and will be $20 million otherwise. Obviously, if the plant operates as expected, FreshDirect will not consider disposing of the plant at year end. But in the unlikely chance that disaster strikes, and it turns out that it is not efficient for FreshDirect to reconfigure the plant, the company will exercise its option to dispose of the plant by selling it to the beer distributor.

Now that we know the payoff to the abandonment option in both states of the world, that is, $0 and $30 million, the next step is to create a portfolio of the underlying plant and a risk-free bond which will exactly match the payoffs to the abandonment option in both states of the world. Assume a risk-free rate of 5%. We will follow the same steps as we did for the pricing of the call option in the one-period binomial model illustrated earlier in this lecture note.

In the likely state of the world where the Bronx plant performs as intended, the replicating portfolio is:

Eq. 25 $$\$125\,\Delta + 1.05\,B = \$0$$

The replicating portfolio in **Eq. 25** of the underlying Bronx plant and a risk-free bond gives the same payoff of $0 for the abandonment option in the outcome where the plant performs as intended. In the Bronx plant failure state of the world, the replicating portfolio is:

Eq. 26 $$\$20\,\Delta + 1.05\,B = \$30$$

By subtracting **Eq. 26** from **Eq. 25**, we can solve for the two unknowns, Δ and *B*. The solution for Δ is -0.2857 and the solution for *B* is $34.01. This solution shows that to replicate the payoff of the put, the replicating portfolio is short -0.2857 of the asset (underlying plant) and has $34.01 million invested in the risk-free security. The value of the abandonment put must equal the value of this replicating portfolio. At the current period, the value of the replicating portfolio is:

Eq. 27 $$\$5.44 = 100(-0.2857) + 34.01$$

Since the replicating portfolio, in theory, exactly replicates the payoff outcomes to the abandonment option, then due to the law of one price, the value of the abandonment option is also $5.44 million. While we solved above using the two simultaneous equations, alternatively, we could have used the general equations (**Equations 18-20**) to solve for Δ and *B*.

$$\Delta = \frac{P_U - P_D}{A_U - A_D} = \frac{0 - 30}{125 - 20} = -0.2857$$

$$B = \frac{P_D - A_D\Delta}{1 + R_F} = \frac{30 - 20\,(-0.2857)}{1.05} = 34.01$$

There are several important points to note about the above example. The conceptual point is that the option to abandon a project will increase the overall value of the project. Static DCF analysis doesn't allow for this option. We were able to calculate the value of the abandonment option without knowing the discount rate for the calculation of the Bronx plant or the associated probabilities with the two possible outcomes. Rather, those estimates are reflected in the underlying $100 million value of the plant.

The FreshDirect example assumed only a single period rather than multi-periods, as would be the case in the real world. But the objective is simply to convey how to think of a real option and the underlying calculations needed to determine its value. Adding real-world time steps certainly would yield a more accurate measure of the value, but not alter the basic concepts. Last, our analysis assumes that we could have created a replicating portfolio via shorting the Bronx plant. But this is not a realistic assumption; it is not realistic to assume that one could locate a twin plant to short. Rather, our objective is simply to conceptually illustrate real options and the framework by which to analyze them.

I wrote the FreshDirect synopsis during the Spring Quarter 2018. When I wrote the synopsis, I did so merely as an exercise, as opposed to thinking that the Bronx plant might not work out as intended. The actual outcome was a complete disaster. When the plant opened in the summer of 2018, FreshDirect simultaneously shut down its existing plant in Long Island City. The expectation of the new plan was FreshDirect would have 50% increased capacity and a substantial reduction in expenses per ticket item, which would be strongly accretive to existing positive cash flows.

The plant failed to work as intended. The technology platform for FreshDirect's pick-and-packing functions was unable to correctly connect with the customer-facing software. The customer would order bacon and would receive broccoli. Literally none of the orders worked as intended. Due to the massive returns, FreshDirect was losing large sums of money on each order, and the cumulative losses put the firm in a massive liquidity crunch. The existing investors, led by the Ackerman family and JP Morgan, were forced to make two sizeable equity contributions to weather the technology disruption. But it wasn't enough. Attempts to sell FreshDirect to Amazon and Walmart were fruitless, even though both firms made substantial offers to buy the company before the new plant became operational.

A year after opening, the plant still wasn't functional. Existing investors capitulated and walked away. New investors provided senior funding, in effect, a prepackaged bankruptcy. Shortly after the original investors stepped aside, COVID-19 hit, just around the time that the company was able to solve its technology issues. Recovery was immediate due to the extraordinary demand for online grocery shopping, coupled with the new plant now working as intended. Soon after, in November 2020, Ahold Delhaize and Centerbridge Partners announced the purchase of FreshDirect at several times what the new investors had put into the company.

In the two above illustrations, we focused on the option to expand and on the option to abandon. We evaluated the expansion option via Black-Scholes and the abandonment option via the Binomial Option Pricing Model. Recall that in the introduction to this lecture note, we considered the option to delay an investment, that is the Bang Bang investment, but we modeled that only via a decision tree and didn't calculate option values per a formal option framework.

Investors must be compensated to bear risk, specifically systematic risk, as one can easily diversify away idiosyncratic risk. But when one accounts for real options, that is the ability to adjust the parameters of a project as learning occurs, idiosyncratic risk becomes a benefit, and the higher the risk the more valuable the option. Just like with a financial option, a real option gives management the right – but not the obligation -- to change or alter an investment. For example, if the cash flow forecast for an investment is all over the board, this level of uncertainty can actually give value, due to the option to delay the investment, for instance.

Based on recent surveys, roughly 30% of large corporations state they employ real options in their valuation of projects. These users tend to be industry specific -- for example, drug companies looking to invest in R&D, and oil companies deciding when to drill. Moreover, they may largely use the framework to help them in recognizing the option, etc., without drilling too deep into the black box as the assumptions can become overwhelming. Rather, when it comes to real options, the first order effect is to recognize the real option and be able to understand the factors which influence the option value. But knowing the inputs with a high degree of certainty is an impossible task.

REAL OPTIONS AND FIRM VALUATIONS

This lecture note has focused on how optionality increases the value of a project. The optionality is multi-faceted, as it can be used to expand, delay, terminate or otherwise alter the project. This section provides a brief overview of how to think about real options in the context of the overall valuation of corporations.

VALUE and GROWTH

This brief overview examines two fictional firms, VALUE and GROWTH, to illustrate the concept of real options in the context of a firm's value. From an asset pricing or investments perspective, value stocks are viewed as stocks which tend to trade at low prices relative to fundamentals such as dividends, cash flow, or earnings. Growth stocks are stocks which have high earnings potential. Some commentators claim that value stocks tend to be cheap and growth stocks the opposite. Below, we frame VALUE and GROWTH firms from a corporate finance perspective and afterwards connect to their meaning with respect to asset pricing or investments.

Consider a firm, VALUE, with assets-in-place -- that is, an operating plant, sales force, etc. To simplify, assume VALUE has constant and perpetual expected cash flows of $100 million annually, and distributes all equity cash flows to the shareholders via dividends and share repurchases. Assume VALUE has just distributed last year's equity cash flow to its shareholders. The cost of capital for VALUE is 8%.

Given the above assumptions, the value of VALUE's assets-in-place is:

Eq. 28
$$\$1.25 \text{ billion} = \frac{\$100 \text{ million}}{0.08}$$

As shown below, VALUE's assets-in-place are displayed on VALUE's balance sheet. Assume that VALUE has no other assets.

VALUE: Market-Value Balance Sheet

Assets-in-Place	1,250	Debt	200
		Equity	1,050
Total Assets	1,250	Total Debt & Equity	1,250

We don't have enough information to indicate whether management of VALUE has created shareholder wealth. That is, VALUE may have undertaken strictly +NPV projects, strictly –NPV projects, or a combination of positive and negative NPV projects, which resulted in the $1.25 billion value of the assets-in-place.

GROWTH also has assets-in-place valued at $1.25 billion, the same as VALUE. But GROWTH has market equity of $1.8 billion, rather than $1.05 billion. We can label the difference of $750 million as *present value of growth options*, or PVGO. The balance sheet of GROWTH is shown below.

GROWTH: Market-Value Balance Sheet			
Assets-in-Place	1,250	Debt	200
Growth Options.	750	Equity	1,800
Total Assets	2,000	Total Debt & Equity	2,000

As mentioned above with respect to VALUE, we have not provided sufficient information to assess whether management created shareholder value regarding the assets-in-place. The focus is to understand the logic behind the $750 million estimate of growth options. The $750 million of PVGO reflects investors' expectations today of NPVs associated with GROWTH future projects. Put differently, investors expect that GROWTH management will undertake new projects which generate a return in excess of the respective cost of capital of such projects. The estimate simply captures the NPV part of the expected future projects, not the total expected value. For example, suppose investors expect GROWTH to invest in numerous future projects, but these projects will generate zero NPV in aggregate -- that is, the return will simply equal the cost of capital. In this case, the value of PVGO is zero and GROWTH has the same value as VALUE.

Adobe Inc. versus Kroger

Adobe Inc. (previously known as Adobe Systems) is a large computer software firm which is widely known for Photoshop (introduced in 1989) and PDF (introduced in 1993). We can think of Photoshop, PDF, and other well-known products such as Adobe Reader and Adobe Creative Suite, as representation of Adobe's assets-in-place. In 2018, Adobe generated roughly $3 billion in cash flow. Think of these cash flows as generated by Adobe's current assets-in-place. Using a cost of capital of 10.8% and assuming perpetual cash flows with zero growth, the value of Adobe's assets-in-place is:

Eq. 29
$$\$\ 27.8 \text{ billion} = \frac{\$3 \text{ billion}}{0.108}$$

And if we assume that Adobe's existing assets-in-place are sufficient to generate cash flows which grow at a rate of 3.0% annual into perpetuity, the value of the assets-in-place increase to:

Eq. 30
$$\$\ 38.5 \text{ billion} = \frac{\$3 \text{ billion}}{0.108 - 0.03}$$

In both cases, the zero growth case and the 3% annual growth case, the value of the assets-in-place is less than a third of the total $120 billion market value of Adobe Inc. As shown in the table below, the implied value of Adobe's PVGO is $92.2 billion under the zero growth case. As noted earlier, these estimates don't reflect the future value of new assets and businesses created by Adobe; rather these estimates reflect the NPV, that is, the expected value created, via future investments.

	Adobe Inc.	Kroger
Enterprise Market Value	120.0	37.0
Assets-in-Place (0% growth)	27.8	47.1
Present Value Growth Options	92.2	-----

One can make the argument that the existing assets-in-place are worth more, and thus the expected growth rate is far higher than assumed here. But it is practically impossible for static assets to perpetually grow at a rate which exceeds the overall growth of the economy. To grow at a higher rate requires subsequent CAPX, which in turn reduces the corresponding near-term cash flows. In any event, the subsequent CAPX in enhancing the assets-in-place would be reflective of the exercise of the growth options.

Kroger is the largest supermarket chain by revenue in the United States. Kroger operates 2,765 supermarkets and multi-department stores. Kroger supplies its supermarkets via 38 food processing and manufacturing facilities. Kroger is subject to intense competition from other large supermarket chains, independent grocery stores, Walmart, which carries grocery products within its hypermarkets (or Supercenters), online grocers, restaurants, and restaurant delivery, etc.

In 2018, Kroger generated roughly \$4.0 billion in cash from its assets-in-place. Using a cost of capital of 8.5% and perpetual zero-growth cash flows, the estimated value of Kroger's assets-in-place is \$47.1 billion:

$$\boxed{\text{Eq. 31}} \qquad \$\,47.1 \text{ billion} = \frac{\$4.0 \text{ billion}}{0.085}$$

If we assumed a positive growth rate, the estimated value of Kroger's assets-in-place would be even higher. However, the overall enterprise market value of Kroger is \$37.0 billion, roughly \$10.1 billion less than the estimated value of the assets-in-place. At first glance, one might conclude that the market is undervaluing Kroger, given that its market enterprise value is less than that of its assets-in-place. But that is not the point of this exercise. Indeed, the most likely explanation is that the cash flows generated by the assets in place are expected to decline over time, rather than stay constant, as assumed above. Another possibility is that the cost of capital is much higher than 8.5%, which also results in a lower valuation of the assets of place.

Notwithstanding any adjustments, large or small, to the various assumptions, it is clear that Adobe derives the majority of its market value from the expected NPV of future real options, whereas Kroger derives its value from existing assets-in-place. From an investments framework, an analyst may view Adobe as a rich security since it has an Enterprise Value / Cash Flow multiple of 40.0 versus a multiple of less than 10.0 with respect to Kroger. Indeed, cash flow multiples are one of the benchmarks used to distinguish value stocks from growth stocks. But from a corporate finance perspective, we are not making a judgement call on cheapness or richness of a company's valuation. Rather we are showing that Kroger's value simply derives from its assets-in-place, and Adobe's value derives from expected profits of new ventures.