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Introduction to Capital Structure: M&M Propositions

"My equity investors expect me to generate returns in the high teens and are very demanding especially during quarterly calls. But I can go out and raise debt and pay only a 5.5% coupon. Thus, debt financing is far cheaper than equity."

INTRODUCTION

Analysts value firms largely by focusing on the assets and projects on the left-hand side, the asset side, of the balance sheet. The value of a project is the sum of the project's expected future cash flows discounted at the project's cost of capital. For a firm with multiple projects, the overall firm value is equal to the sum of the value of all projects, other assets including excess cash, plus growth options not associated with the projects.

A common assumption, at least in many corporate finance textbooks, is that management can quickly raise funds for projects expected to yield +NPV -- that is, that will create shareholder wealth. As long as the project is expected to generate positive +NPV, financing is readily available, even if positive cash flows are not likely to accrue for several years. Notwithstanding this assumption, it is useful to build a framework by which to understand the financing of corporations. In the simplest terms, the corporation issues debt (bank loans or bonds) and equity (various types of stock) to investors. In return, these investors have a claim on the assets of the firm, namely a claim to the expected future cash flows. In the case of an all-equity firm, all expected future cash flows belong to the shareholders. For firms with debt in addition to equity, the debtholders are entitled to receive a fixed payout, and the shareholders receive the residual cash flows.

The bifurcation of cash flows into straight debt and equity is obviously simplistic, as corporations issue all types of securities. However, this straightforward bifurcation allows for a solid understanding of real-world capital structure and paves the way for the subsequent discussion of complex securities.

Can management take advantage of financing opportunities and realize +NPV on the right-hand side of the balance sheet, just like with real assets on the left-hand side of the balance sheet? Suppose the cost of long-term debt financing at Procter & Gamble is 4.6% and the cost of equity financing is 9.0%. Is it possible for Procter & Gamble to lower its overall cost of capital, and thereby increase firm value, by increasing its level of debt financing and decreasing its equity financing? Or can Procter & Gamble instead create value by reducing the relative amount of debt financing, since debt requires fixed and periodic payments to the debtholders, whereas dividend payments to shareholders are not mandatory? Framed more generally, what is the mix of debt and equity that produces the highest firm value? There are two primary questions. First, as framed by the question above, can management alter the firm's cost of capital by merely rearranging the mix of debt and equity? Second, what mix of debt and equity generates high operating efficiencies, low bankruptcy costs, minimal managerial malfeasance, etc.? That is, is there a debt/equity ratio that is optimal for creating value on the left-hand side of the balance?

MODIGLIANI & MILLER

Consider Procter & Gamble, a multinational manufacturer and marketer of consumer products. Procter & Gamble has an equity market capitalization of $360 billion as of July 2023 and a book value of debt of $34 billion. The annual EBITDA for Procter & Gamble is roughly $21.1 billion; it incurs interest expense of around 630 million. Due to the low leverage and high-interest coverage, Procter & Gamble has an investment grade credit rating of AA- and minimal chance of bankruptcy. Prior to the era of modern corporate finance theory, the prevailing wisdom was that since the likelihood of default was so low, Procter & Gamble could simply tweak its capital structure a bit by raising "cheap debt" (4.6%) and repurchasing "expensive equity" (9.0% required return), and create instant value, as this slight adjustment would not impact the probability of financial distress.[1] While this is not the prevailing wisdom today, at least in academic circles, it is still widely held by many practitioners and corporate managers.

In 1958, the *American Economic Review* published a path-breaking paper by Modigliani and Miller entitled "The Cost of Capital, Corporate Finance, and the Theory of Investment."[2] Modigliani and Miller proved that capital structure does not influence firm value. Essentially, management's decision on how to finance the firm is irrelevant. A levered firm has the same value as an all-equity firm, everything else remaining the same. This paper ushered in the birth of modern corporate finance theory as we know it today.

Modigliani and Miller assume perfect capital markets -- that is, a world absent of agency costs, taxes, information asymmetries, transactions costs, and other real-world market imperfections. Rather than assuming the future cash flows are known with certainty, Modigliani and Miller assume expected future cash flows are risky and thus not known with certainty. This assumption of risky cash flows is a significant distinction, as we will discuss further in this lecture note.

Rather than operate in an academic vacuum, Modigliani and Miller recognize the importance of carefully identifying the assumptions, as they allow us to consider the optimal debt/equity ratio in an environment

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[1] Note that the 4.6% debt estimate assumes the issuance of long-term debt as of July 2023. The 1.9% blended interest rate based on the existing level of debt and interest expense is due to Proctor & Gamble raising short term debt when U.S. Treasury rates were lower.

[2] Franco Modigliani and Merton Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, June 1958, pp. 261-297.

free of market imperfections. Once we understand the capital structure implications in the world without frictions, we can relax the restrictive assumptions and assess their impact on real-world capital structure and resulting firm value.

PROPOSITION I

Consider two firms which are identical with one exception, namely, their capital structures are different. One firm, ALLEQUITY, has zero debt. The other firm, LEVER, has a capital structure composed of debt and equity. Imagine these two firms produce the same product, employ the same quality of workers, utilize the same technology, and so forth. ALLEQUITY generates expected annual profits of $50 million with a 10% cost of capital. It has just paid a dividend of $50 million, reflecting the previous year's profits. All of ALLEQUITY's assets in place are required to generate the expected annual profits of $50 million, which are expected to remain constant forever. The value of ALLEQUITY can be represented as follows:

Eq.1 $$V_{ALLEQUITY} = D_{ALLEQUITY} + E_{ALLEQUITY} = E_{ALLEQUITY}$$

or \$500 million = \$0 + \$500 million[3]

LEVER has $250 million in perpetual debentures which pays an annual coupon of 5.0%. Given the condition stated above -- that LEVER is identical to ALLEQUITY other than the difference in capital structure -- LEVER also generates expected profits, before interest, of $50 million annually, forever. And like ALLEQUITY, LEVER will pay out all its profits after interest as dividends. The total firm value and equity value of LEVER is unknown:

Eq. 2 $$V_{LEVER} = D_{LEVER} + E_{LEVER}$$

or V_{LEVER} = \$250 million + E_{LEVER}

Supppose an investor purchases 20% of the shares in ALLEQUITY. What is their payoff?

Investment Strategy: Buy $100 million of stock in ALLEQUITY

Investment	Return
Buy 20% of $E_{ALLEQUITY}$	20% of Expected Profits of $E_{ALLEQUITY}$

The 20% ownership in ALLEQUITY entitles the investor to 20% of the profits (note in this example, profits are equal to cash flows, as there are no taxes, no changes in net working capital, no CAPX, etc.). At the end of each year, the investor expects to receive $10 million in dividends. Suppose the investor chooses to replicate this payoff via purchasing securities in LEVER. Modigliani and Miller show that this can be

[3] The $500 million value of ALLEQUITY is based on applying the perpetuity formula to the expected constant stream of cash flows of $50 million into perpetuity and the 10% cost of capital.

accomplished by buying both debentures and stock in LEVER. In this particular example, the investor will invest $50 million in LEVER debentures and also acquire 20% of LEVER stock.

Replicating Strategy: Invest $50 million in LEVER debentures and 20% of LEVER stock

Investment	Return
Buy 20% of D_{LEVER}	20% of Interest
Buy 20% of E_{LEVER}	20% of Expected Profit – Interest

The replicating strategy yields an identical payoff to the initial investment strategy. Both investment strategies return 20% of total operating profits. The only difference is that in the replicating strategy, the investor receives $2.5 million of interest and $7.5 million of expected dividends.[4] Given this payoff is identical to the payoff of stock in ALLEQUITY, it implies the cost of the replicating portfolio must be identical to the cost of the initial portfolio. Thus, the investor pays $50 million for the 20% of LEVER stock, and hence the value of LEVER is:

Eq. 2a $$V_{LEVER} = \$500 \text{ million} = \$250 \text{ million } D_{LEVER} + \$250 \text{ million } E_{LEVER}$$

This proof is the foundation for Proposition I. Note that their proof was a formal mathematical proof, as opposed to the numerical example provided here.

Proposition I:

"…the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate appropriate to its class…..the average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalization rate of a pure equity stream of its class."

But what if Proposition I does not hold? Suppose for example the value of LEVER is equal to $450 million rather than to $500 million. This would imply:

Eq. 2b $$V_{LEVER} = \$450 \text{ million} = \$250 \text{ million } D_{LEVER} + \$200 \text{ million } E_{LEVER}$$

That is, the value of the debt remains at $250 million, but the equity value is $200 million rather than $250 million. An arbitrageur will immediately purchase 20% of D_{LEVER} for $50 million as above and purchase 20% of E_{LEVER} for $40 million. As indicated above, these purchases entitle the arbitrageur to receive the same payoff as above, namely $2.5 million of interest and $7.5 million of dividends. Moreover, the arbitrageur will also have the excess $10 million to invest elsewhere. Arbitrageurs will buy LEVER, and short sell ALLEQUITY,

[4] Despite purchasing the same percentage of shares, the expected dividends of $7.5 million are less than in the case of ALLEQUITY due to LEVER's payment of interest to the debtholders.

until LEVER had a value equivalent to ALLEQUITY, consistent with *the law of one price*, namely, two assets must have the same price if they yield identical risks and payoffs. If their prices are different, then the arbitrageur can make a risk-free profit, and what is important, in companies which generate risky cash flows. This arbitrage proof paved the way for researchers to set up arbitrage conditions in many other areas of finance, notably option pricing, in which the same principles are applied.[5]

As described above, the investor purchases stock in ALLEQUITY and then creates a replicating portfolio with a purchase of both debentures and stock in LEVER. Alternatively, the investor can purchase shares in LEVER and then create a replicating portfolio via purchasing shares in ALLEQUITY. Assume the investor purchases 20% of the stock in LEVER. Further, assume that the value of LEVER is given as:

Eq. 2c $$V_{LEVER} = \$500 \text{ million} = \$250 \text{ million } D_{LEVER} + \$250 \text{ million } E_{LEVER}$$

And further assume that the value of ALLEQUITY is unknown.

Investment Strategy: Invest $50 million of stock in LEVER

Investment	Return
Buy 20% of E_{LEVER}	20% of Expected Profits – Interest of E_{LEVER}

The expected operating profits of LEVER are $50 million, the total interest payments are $12.5 million, and the expected dividends are $37.5 million. Thus, the 20% investment in LEVER stock yields an expected $7.5 million annually in dividends to the investor. This investment can be replicated via a purchase of stock in ALLEQUITY. The investor can simply borrow an amount equal to 20% of the debt in LEVER and use those proceeds to supplement a 20% purchase of stock in ALLEQUITY.

Replicating Strategy: Borrow an amount equal to 20% of the debt of LEVER and use to partially finance a 20% purchase of ALLEQUITY

Investment	Return
Borrow 20% of D_{LEVER}	-20% of Interest
Buy 20% of $E_{ALLEQUITY}$	20% of Expected Profits of $E_{ALLEQUITY}$

Thus, the investor borrows $50 million and will use this $50 million, along with non-borrowed funds, to buy 20% of ALLEQUITY. Given perfect capital markets, the investor pays the same rate of interest, 5%, as would LEVER.[6] At year-end, the investor receives $10 million in dividends from her shares in ALLEQUITY and

[5] Modigliani and Miller were not the first to use the concept of arbitrage. Rather, it was already well known in international economics with respect to the interest rate parity theorem. The difference is that with interest rate parity, you are starting with securities which are risk free in their respective countries. But Modigliani and Miller were able to create replicating portfolios with assets which were not risk free and hence the huge leap forward in creating modern finance theory.

[6] Modigliani and Miller assume corporate debt is risk-free. Similarly, given perfect capital markets, if investors posted their ALLEQUITY shares as collateral, they can likewise borrow at risk-free rates on the same leverage ratio as LEVER. Recall that the lender to the investor will not be concerned about information asymmetry, investor malfeasance, etc. While these assumptions don't necessarily apply in the real world, individual investors can borrow in their retail

simultaneously pays $2.5 million in interest, for a net payoff of $7.5 million. That is, the replicating portfolio using borrowed funds to buy stock in ALLEQUITY yields an identical payoff to the purchase of stock in LEVER. It follows, therefore, that if the payoff is the same, and the underlying riskiness of the asset cash flows are the same, then the cost of the investments should be identical. In other words, the investors pay $100 million for the 20% share in ALLEQUITY stock. Thus,

Eq. 3 $$V_{LEVER} = V_{ALLEQUITY}$$

Again, the conclusion is the same. The value of a firm with debt is identical to an all-equity firm with the same structure of cash flows, assuming perfect capital markets. Given investors can create replicating portfolios with identical payoffs, the law of one price holds. If the firm values are different, arbitrageurs instantaneously buy stock in one firm and short stock in the other firm to realize risk-free profits.

In proving that capital structure does not matter in perfect capital markets, Modigliani and Miller built the foundation for modern corporate finance theory. As a result, we now know that if capital structure impacts firm value, it is because some of the assumptions in their model do not hold. Modigliani and Miller were also the first researchers to construct replicating portfolios with risky assets. You can think of their replication as that of static replicating portfolios, as the replicating portfolio has the same cash flows as the underlying portfolio. Indeed, the famous put-call parity relationship is just Proposition I of Modigliani and Miller.[7] What they followed with is known as *dynamic replication,* e.g., Black-Scholes option pricing where the replicating portfolio doesn't have the same cash flows. Instead, for very small changes in the parameters, the price of the underlying asset and the replicating portfolio will change by the same amount (thus, it has the same "greeks" at a specific point).

Proposition II directly follows from Proposition I. As stated by Modigliani and Miller,

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Proposition II:

"From Proposition I, we can derive the following proposition concerning the rate of return on common stock in companies whose capital structure includes some debt: the expected rate of return or yield, i, on the stock of any company j belonging to the kth class is a linear function of leverage as follows:

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brokerage accounts at margin rates slightly higher than fed funds rates -- in some cases, only 30 basis points higher. They can do so as a result of posting their shares as collateral, thereby allowing the brokerage firm to take possession in case they become at risk of a loss. And just like the fact that shareholders face limited liability in the corporate form of organization, individual investors can also engage in a private contract whereby they borrow funds, non-recourse (backed by collateral, such as securities, but with no further recourse from the borrower).

[7] The lecture note, *Agency Costs and Capital Structure*, describes the context of the put-call parity relationship with respect to capital structure.

$$i_j = p_k + [p_k - r]\frac{D_j}{S_j}$$

That is, the expected yield of a share of stock is equal to the appropriate capitalization rate p_k for a pure equity stream in the class, plus a premium related to financial risk equal to debt-to-equity ratio times the spread between p_k and r."

To illustrate, reconsider ALLEQUITY. The value of ALLEQUITY is $500 million. Assume management of ALLEQUITY chooses to substitute $100 million of debt, at an interest rate of 5%, for equity in its capital structure. As proved by Proposition I, the enterprise value of ALLEQUITY, newly named LEVER, stays constant at $500 million. Proposition II, as shown above, shows what happens to the expected return to equity as a result of the recapitalization. Table 1 illustrates the recapitalization.

Table 1

	ALLEQUITY	LEVER
Number of Shares	25 million	20 million
Stock Price	$20	$20
Equity Value	$500 million	$400 million
Debt Value		$100 million
Expected Operating Income	$50 million	$50 million
Interest		$5 million
Equity Earnings	$50 million	$45 million
Earnings Per Share	$2.00 per share	$2.25 per share
Expected Return on Shares	10.00%	11.25%

Simultaneous with the new debt issuance of $100 million at a 5% coupon rate, ALLEQUITY repurchases 5 million shares at the market share price of $20. Per Proposition I, firm value remains the same, despite the changes in the capital structure, and thus there is no change in the share price. Since the adjustment in the capital structure doesn't influence the left-hand side of the balance sheet denoting the assets of the firm, the expected operating income remains constant at $50 million. Due to the interest payment of $5 million, the equity earnings of LEVER are $45 million, versus $50 million for ALLEQUITY. And expected earnings per share, or EPS, increases from $2.00 to $2.25. For ALLEQUITY, the expected return on shares is:

$$\text{ALLEQUITY Expected Return on Shares} = 10.00\% = \frac{\$50 \text{ million}}{\$500 \text{ million}}$$

the expected return on shares for LEVER is:

$$\text{LEVER Expected Return on Shares} = 11.25\% = \frac{\$45 \text{ million}}{\$400 \text{ million}}$$

Thus, the expected return on equity increases with an increase in the leverage ratio, as indicated above by Proposition II. And we know from Proposition I that the average cost of capital of a firm can be given as:

Eq. 4

$$R_A = R_D\left[\frac{D}{[D+E]}\right] + R_E\left[\frac{E}{[D+E]}\right]$$

As covered in *Investments*, the expected return on a portfolio is equal to the weighted average of the expected returns on the various positions in the portfolio. And in the context of a firm, the expected return to the firm is the weighted average of the expected return to the various securities of the firm, and in the case of **Eq. 4**, debt and equity. Solving **Eq. 4** for the expected return to equity yields:

Eq. 5

$$R_E = R_A + [R_A - R_D]\frac{D}{E}$$

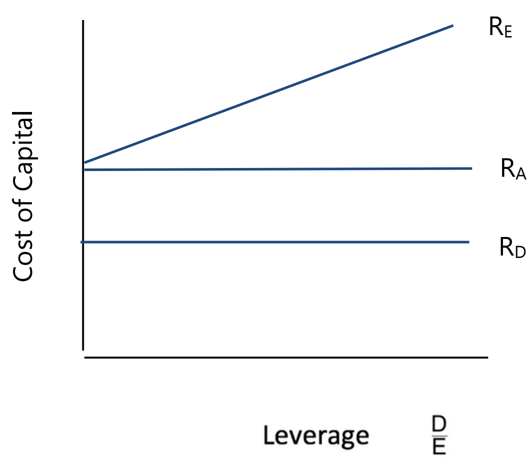
And with respect to the recapitalization illustration above:

Eq. 5a

$$11.25\% = 10.00\% + [10.00\% - 5.00\%]\frac{100}{400}$$

Eq. 5a yields the same answer for the expected return to equity as in Table 1, above. Thus, as per the original quote from Modigliani and Miller, Proposition II shows that the expected return to equity increases linearly with leverage. And as indicated earlier, Modigliani and Miller assumed in their proofs of Proposition I and Proposition II that debt is risk-free, hence the linear relation between the expected return to equity and increases in the debt-to-equity ratio. Given the firm value and the share price do not change, and the expected return on equity increases, it follows of course that the equity risk increases commensurate with the increase in the expected return. Put differently, when a firm increases its leverage, it amplifies the volatility of the return stream to the equity holders, who in turn, demand a higher return to hold the shares.

Figure 1

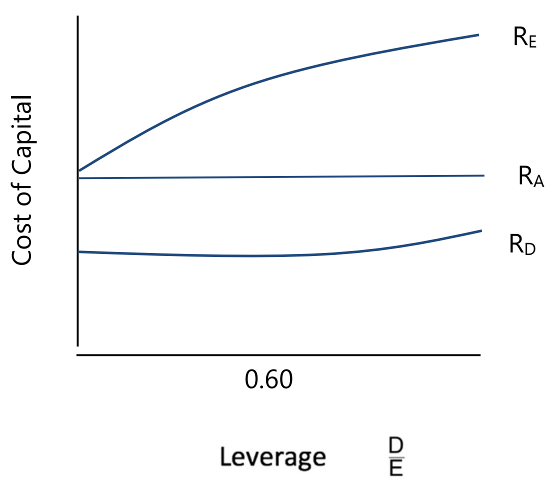


As discussed in footnote 6, Modigliani and Miller assumed that debt was risk-free. They did so primarily to sharpen the distinction between risky equity and risk-free debt. But it did not drive the proofs. They were challenged in a big way in that an asset pricing model, such as CAPM, did not exist when they formalized their proofs, which is all the more impressive given their paper's accomplishments. As there was no robust concept of a market risk premium, Modigliani and Miller took the clever tack of assuming the two firms had the same business risk, and thus were in the same risk class.

Several years after the development of the CAPM, Rubinstein (1973) merged the CAPM with the Modigliani and Miller capital structure propositions; it was able to allow for risky debt via the CAPM.[8] Moreover, where Modigliani and Miller generally assumed that the firms were in the same risk class, Rubinstein replaced this notion with that of the firms having the same underlying asset beta. In effect, Rubenstein was then able to prove the Modigliani and Miller theorems via a mean-variance framework, namely that of the CAPM.

Conceptually, when debt becomes risky, it starts to bear systematic business risk -- that is, the beta on corporate debt goes from zero to positive. Obviously, firms with highly risky operations, such as firms with large fixed costs in cyclical industries, will have risky debt if their leverage ratios are sufficiently high. As leverage causes the debt to become risky, debtholders will demand a higher return to compensate them for the added risk; in other words they will demand a credit risk premium. Figure 2 below extends Figure 1 to include risky debt.

Figure 2



For purposes of illustration only, assume here that debt becomes risky once the proportion of debt in the capital structure exceeds 60%. As shown in Figure 2, the expected return on debt is constant and risk-free when the proportion of debt in the firm is 60% or less, and the expected return on equity increases linearly with increasing leverage. But when the proportion of debt exceeds 60%, the required return on debt

[8] Mark Rubenstein, "A Mean-Variance Synthesis of Corporate Financial Theory," *Journal of Finance* (March 1973). Rubinstein is also well known for his development, with Hayne Leland, of portfolio insurance, which played a major role in the market crash of October 1987.

increases because the debtholders are not expected to be paid in full if a corporate default occurs. The expected return on equity continues to increase as its risk increases, as leverage rises. This is because if default occurs, the creditors receive payment before the shareholders receive their claims. But note that as the expected return on debt increases, due to the overall business risk of the firm, the equity risk begins to increase at a decreasing rate. While the expected return on debt and equity are increasing, the overall cost of capital remains fixed due to the relative proportion of the debt increasing and the relative proportion of the equity decreasing. At the limit, where the firm is completely leveraged, with 100% debt, (as in the case of a bankruptcy, whereby the debtholders receive all of the future cash flows, the expected return on debt becomes the overall cost of capital. Indeed the firm reverts to an all-equity capital structure. That is, bankruptcy occurs, but the formality of bankruptcy itself is not costly, as all that happens is that the equityholders turn over the keys to the debtholders.

Recap of Propositions I and II

Like many excellent academic papers, the work of Modigliani and Miller was not instantly embraced. In fact, it was met with a lot of skepticism, if not criticism.[9] The critics challenged the assumptions for being unrealistic, and for the concept that arbitrage could work with risky assets. At the time, interest rates were low and expected returns on equity were high, such that there was a large spread between expected equity returns and debt returns. The prevailing wisdom was that a firm with no or minimal debt could increase leverage a bit, automatically creating value due to the expected return on equity not increasing as much as predicted by Modigliani and Miller. Moreover, this belief was not simply based on the ability to deduct interest expenses with respect to taxes. Instead, it was a view that management could somewhat create value on the right-hand side of the balance sheet via financial engineering. That is, it was based on the understanding that a firm like Procter & Gamble with a strong Investment Grade bond rating and a low level of leverage can increasec (or create) shareholder value via a slight amount of additional leverage. But this understanding missed the point of Proposition II: that increases in leverage, however slight, result in high costs of equity capital.

The publication of the paper resulted in two Nobel Prizes, with Modigliani being awarded the prize in 1985, and Miller in 1990. The paper created what today is known as modern corporate finance theory. The subsequent academic work has used Modigliani and Miller as the foundation of corporate finance, and has focused both empirically and theoretically on how capital structure impacted cash flows and investors' expectations of cash flows based on the relaxation of the various assumptions underlying the theory.

[9] Indeed, Merton Miller's view was that they had to first convince themselves that their proofs were correct. Miller was on the Chicago Booth faculty for nearly 40 years and was considered the intellectual leader of the school until his passing in 2000. Miller was a great friend and colleague of mine, and we had just compleed the first draft of a chapter for a book of academic papers about market crashes when he passed.